Diffuse Solar Irradiance and Atmospheric Turbidity

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The measurement of diffuse solar irradiance along with global and direct solar irradiance provides a valuable key to understanding the properties of the atmosphere. This information is of value to meteorologists and climatologists as well as to designers of solar thermal or photovoltaic energy systems. As an additional benefit this information can be correlated to meteorological visibility and thus be of benefit to many sectors of society. This paper describes the practical measurement of diffuse, global and direct solar irradiance. The relationship of diffuse irradiance to the Linke turbidity factor and solar elevation angle is examined. Important concepts related to visibility are defined, and we conclude by illuminating the relationship between measurements of diffuse solar irradiance on clear days and meteorological visibility.

1. Introduction

Pyranometers are used to measure the global solar irradiance I_G on a horizontal surface, and hourly, daily and monthly data is available from meteorological services in many parts of the world. Diffuse irradiance I_F on the horizontal can be achieved using a shadow ring such as the Kipp-Zonen 121B which is shown in Figure 1. A pyranometer mounted on the axis of the ring receives only diffuse radiation, for the ring is adjusted regularly to ensure that the direct irradiance does not reach the pyranometer. In a time series of measurement at a specific position (latitude *L* and longitude *M*) the corresponding solar elevation angle *v* can always be found [1].

Figure 1: *A shadow ring and pyranometer are used for measurement of the diffuse solar irradiance.*

Armed with the knowledge of the global and the diffuse irradiance, it is possible to compute the direct irradiance I_{DH} striking the horizontal, for $I_G = I_F + I_{DH}$. The quantity I_{DH} is related to the direct beam irradiance I_D measured normal to the direction of the direct rays of the sun by $I_{DH} = I_D \sin v$. Major meteorological observatories and other institutions may also measure the direct solar irradiance directly by mounting a broad band (300-4000 nm) pyrheliometer on a tracker which follows the sun, as shown in Figure 2. The measurement of diffuse irradiance is a relatively simple measurement, while direct solar irradiance measurements require a tracker and associated support infrastructure.

2. Atmospheric extinction and Linke's turbidity factor

Light passing through the atmosphere is scattered and absorbed by molecules and particles. *Scattering* refers to processes in which photons change direction after an interaction, while *absorption* occurs when photons are removed from a beam of light, and their energy is converted to an excitation of atoms or molecules. Light also interacts with larger particles such as water droplets and dust when these are present in the air. Rayleigh scattering due to interactions with air molecules is well understood and provides a good description of the behavior of light in a perfectly clean, dry atmosphere. Mie scattering theory is used to describe large (>10 nm) particle scattering. Molecular absorption occurs within specific wavelength bands. Some examples are Hartley, Huggins and Chapuis absorption bands in the ultraviolet C and B bands and the visible region due to the trace gas ozone. Oxygen, water vapor, nitrogen dioxide and carbon dioxide are also responsible for some important absorption phenomena.

Figure 2: *Tracker with pyrheliometers used to measure direct solar irradiance I_D in northern Greenland (76,5^{O} N) [2].*

The combined effects of scattering and absorption is referred to as *extinction* and can often be described by means of the Lambert-Beers exponential attenuation law which for the case of the direct solar beam radiation on a clear day can be expressed using the following equation:

$$I_D(m) = 1367 \cdot F \cdot \exp[-0.8662 \cdot T_L \cdot m \cdot D_R(m)] \qquad (1)$$

$I_D(m)$ is the beam irradiance as a function of the air mass m through which the beam has traveled, 1367 (W/m^2) is the mean solar irradiance outside the atmosphere (the "solar constant"), F is a factor which takes account of the yearly variation in the earth-sun distance, T_L is the Linke turbidity factor, and $D(m)$ is the optical depth due to pure Rayleigh scattering as a function of the air mass. In an ideal Rayleigh atmosphere $T_L = 1$. The closest to this ideal value (T_L 2) is achieved in clear, cold air at high latitudes, particularly in Antarctica, but also in pristine Arctic locations such as Northern Greenland.

In order to calculate values of $I(m)$ at a specific date and time at a specified location on the earth's surface, one must be able to compute $D(m)$ for the pure Rayleigh atmosphere. A very useful empirical equation has been developed by Louche, Peri and Iqbal and modified by Fritz Kasten [3]:

$$\frac{1}{D_R(m)} = 6{,}6296 + 1{,}7513 \cdot m - 0{,}1202 \cdot m^2 + 0{,}0065 \cdot m^3 - 0{,}00013 \cdot m^4 \qquad (2)$$

For large solar elevation angles (> 30°) the air mass $m = 1/\sin(v)$, where v is the solar elevation angle. A more precise relationship valid for air masses up to about air mass 6 (and thus for rather low solar elevation angles) is the following equation developed by F. Kasten and A.T. Young [4]:

$$m = \frac{1{,}002432\ \sin^2 v + 0{,}148386\ \sin v + 0{,}0096467}{\sin^3 v + 0.149864\ \sin^2 v + 0.0102963\ \sin v + 0{,}000303978} \qquad (3)$$

For a non-ideal atmosphere with particulates, haze and other impurities, the Linke turbidity factor can provide a good estimate of the direct beam radiation reaching the surface of the earth. Table I shows typical values for T_L.

Atmospheric Properties	Linke Turbidity Factor
Pure Rayleigh atmosphere	1
Extremely clear, cold air (arctic)	2
Clear, warm air	3
Moist, warm air	4-6
Polluted atmosphere	8

Table I: *The Linke turbidity factor under typical atmospheric conditions [4].*

Equations (1), (2) and (3) can be used to compute the direct solar irradiance in radiation models designed to predict the performance of solar thermal or photovoltaic energy systems. Table II shows the results of a calculation of the direct solar irradiance at midsummer in Denmark with a solar elevation angle $v = 57°$:

T_L	1	2	3	4	5	8
I_D(W/m²)	1173	1039	921	817	774	504

Table II: *Equations (1)-(3) were used to compute the direct solar irradiance at noon at midsummer in Silkeborg, Denmark (56°N).*

If the atmosphere were perfectly clear ($T_L = 1$), then the direct irradiance in this case would be 1173 W/m^2. Typically the Linke turbidy factor on a clear Danish summer day is about 3, and values around 921 W/m^2 are indeed observed in practice.

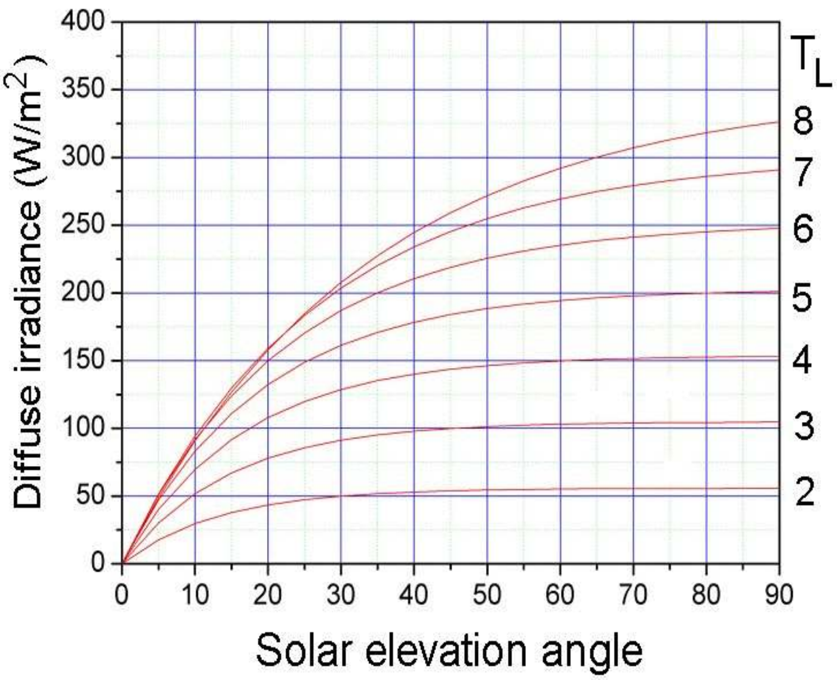

Figure 3: *The observed diffuse irradiance on a horizontal surface depends upon the value of the Linke turbidity factor and the solar elevation angle.*

The greater the atmospheric turbidity, the greater the diffuse irradiance I_F observed on a horizontal surface. From the European Solar Radiation Atlas [4] we have the useful data in Figure 3 showing the diffuse irradiance on a horizontal surface corresponding to a range of values for Linke's turbidity factor.

In a practical situation one will know the time and location and thus have access to the solar elevation angle *v*. It neither difficult nor expensive to measure the diffuse solar irradiance I_F on a horizontal surface. Therefore, it is desirable to find the Linke turbidity factor expressed as a function of the two known variables: the solar elevation angle and the diffuse irradiance. This program of mathematical analysis has been carried out and the results are shown in Figure 4 [5].

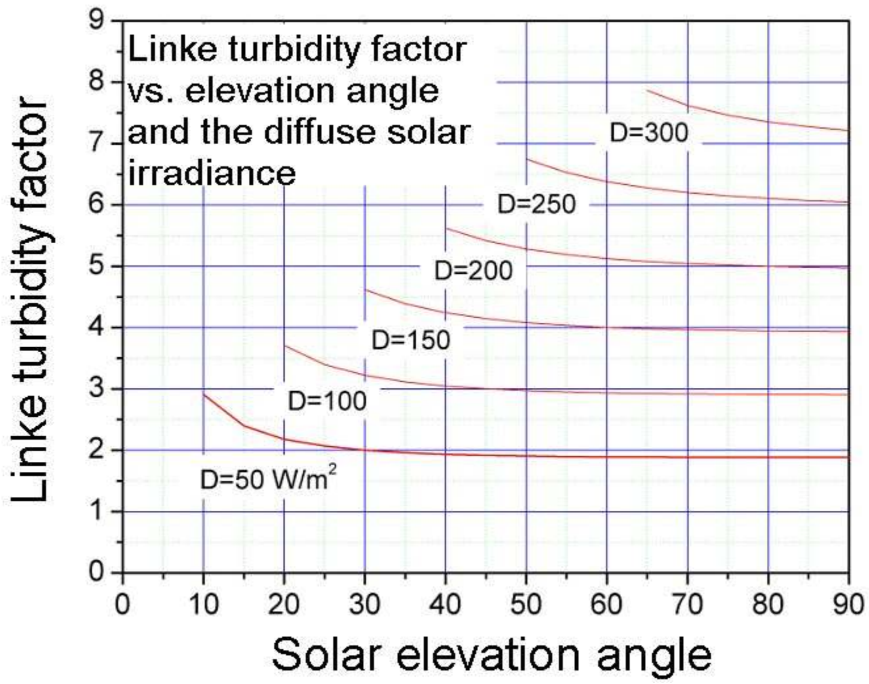

Figure 4: *A matematical inversion of the data of Figure 3 is displayed here. The Linke turbidity factor can be found from the elevation angle and the diffuse irradiance on a horizontal surface.*

3. Meteorological range and visibility

The goal of this paper is to investigate the connection between the Linke turbidity factor and the *meteorological range R*. Pursuant to this end we provide a brief summary of the definition of meteorological range.

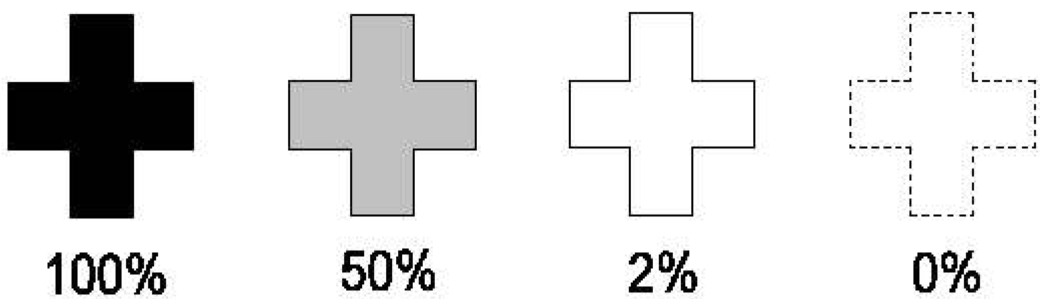

Figure 5: *The contrast of a black target on a white background depends upon the perceived luminance of the target and the background against which it is viewed.*

The *contrast* C perceived by an observer is defined by:

$$C = \frac{L_B - L_D}{L_B} \tag{4}$$

where L_B is the luminance of the background and L_D is the luminance of an object viewed by an observer some distance away. The apparent luminance of the object (e.g. a black cross) increases with distance due to scattering of photons into the line of sight of the observer. The luminance depends upon the distance x of the observer from the object and on the atmospheric extinction coefficient k as follows:

$$L_D = L_B \cdot (1 - e^{-kx}) \tag{5}$$

Substituting L_D into equation (4) yields:

$$C = \frac{L_B - L_D}{L_B} = \frac{L_B - L_B \cdot (1 - e^{-kx})}{L_B} = e^{-kx} \tag{6}$$

If the reference object is not completely black but has contrast C_0 at close range, then $C = C_0\, e^{-kx}$.

The *meteorological range* $x = R$ is defined as the distance from the target at which the contrast viewed by the observer is reduced to 2% = 0,02. For a completely dark object this means that $e^{-kR} = 0.02$. The equation can be solved to find the connection between the meteorological range and the atmospheric extinction coefficient:

$$R = \frac{-\ln(0.02)}{k} = \frac{3.912}{k} \tag{7}$$

This relationship is well known and termed *Koshmeider's formula* [6]. It is important because it links the extinction coefficient directly to the visibility. Thus if the reported meteorological range at an airport is 10 km, then the extinction coefficient k would be $3.912\ 10^{-4}\ \text{m}^{-1}$.

4. Diffuse solar irradiance and visibility

Addressing now the main goal of the paper, we will investigate how measurements of the diffuse solar irradiance and knowledge of the solar elevation angle on a clear day permit an estimate of the meteorological visibility. To make this connection the following strategy can be applied:

(1) Compute the Linke turbidity factor using the algorithm presented for T_L using known values of the clear day diffuse irradiance on the horizontal I_F and the solar elevation angle v. See Figure 4.

(2) Use the Linke turbidity factor to compute the exponent K of the exponential function in Equation (1):

$$K = -0.8662 \cdot T_L \cdot m \cdot D_R(m) \tag{8}$$

(3) The optical depth m is related to the distance traveled from the "top" of the atmosphere by the direct beam radiation by $x = x_S\, m$ where x_S is the *scale height* of the atmosphere. For a standard atmosphere x_S is about 8.0 kilometers. Thus

$K = k \; x_S$ *or* $k = K/x_S$.

(4) Solving for the extinction coefficient:

$$k = -0.8662 \cdot T_L \cdot (m/x_S) \cdot D_R(m) \qquad (9)$$

(5) Once k has been determined, the meteorological range R can be found from Koshmeider's formula.

$$R = \frac{-\ln(0.02)}{k} = \frac{3.912}{k} \qquad (10)$$

Note that this method assumes uniformity of the air mass through which the solar beam radiation passes up to the scale height. In fact most of the scattering effects which contribute to atmospheric turbidity occur within a few kilometers of the surface of the earth. The observant air traveler taking off or landing on a clear day will have noticed the characteristic boundary layer where most atmospheric aerosols are present. The boundary layer is typically about 1000 meters deep with heavier aerosols gravitating towards the bottom of the layer.

The atmosphere is most dense close to the surface, so that the Rayleigh scattering contribution is also due primarily to the lower atmosphere. The visibility property we seek is characteristic of the thin layer of air near the surface. Visibility is also important higher up, as slant range visibility is crucial to aircraft operations during takeoff and landing. Forward scattering due to aerosols is substantially greater when facing a light source (the sun) than when facing away from it, so this must also be taken into account when relating this method to visibilities observed at the surface or in the air.

5. Sample computations

Table IV shows the results of typical computations based on diffuse and direct solar irradiance measured in Silkeborg, Denmark at 56.1 N latitude.

CASE	Diffuse irradiance	Elevation angle	Linke turbidity	D_R(m)	Extinction (m^{-1})	Range (km)
23 mar	55 W/m^2	34.8	2.07	0.107	4.185E-05	93.5
25 apr	200 W/m^2	46.6	8.02	0.113	1.349E-04	29.0

Table IV: *These data were collected in Silkeborg on clear days around noon. The first day was exceptionally clear, the second was hazy.*

6. Simplified equation

The equations of steps (4) and (5) can be combined to yield a simple connection between the Linke turbidity factor and the visibility:

$$k = -0.8662 \cdot T_L \cdot (m/x_S) \cdot D_R(m) = \frac{3.912}{R} \qquad (11)$$

Recall that the scale height $x_S = 8.0$ *km* and that $m = 1/\sin(v)$ for elevation angles above about 30 degrees. Note also that the Rayleigh optical depth term $D_R(m)$ is not strongly dependent on the air mass but varies from around 0.10 to 0.12 for the elevation angles of interest. This can be set equal to 0.11 for this approximation. Solving for the range R one thus obtains:

$$R \approx 328 \cdot \left(\frac{1}{T_L}\right) \cdot \sin(v) \qquad (12)$$

Inserting the data from the first case in Table IV: *R = 90.4 km* which is close to the value obtained using the complete algorithm. Agreement is within 5-10% in most cases. It would in future work be of interest to obtain more extensive data series with corresponding values of the surface visibility and the turbidity factor on clear days to provide validation for this model.

7. Conclusion

We have reviewed the definition of the Linke turbidity factor T_L and its use to predict the diffuse irradiance on a horizontal surface for a range of solar elevation angles. An algorithm has been developed to invert the usual definition so that values of the solar elevation angle and the diffuse irradiance yield the value of the turbidity factor. The turbidity factor along with the air mass m and the Rayleigh optical depth, which is a function of m, can be used to find the atmospheric extinction coefficient for the slant path of the direct rays of the sun. The definition of meteorological range R is related by the Koshmeider equation to atmospheric extinction when light passes from an object to an observer in the lower part of the atmosphere. We have attempted to find a useful relationship between the turbidity factor and the meteorological range.

8. Acknowledgements

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